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Assessing uncertainties in balanced cross sections

Phoebe A. Judge, Richard W. Allmendinger*

Dept. of Earth & Atmospheric Sciences, Cornell University, Ithaca, NY 14853, USA

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ABSTRACT

Balanced structural cross sections are models that are fit to incomplete data. The models are under-constrained with respect to any particular two-dimensional line-length model, but enough data generally exists to yield a well-constrained area balance solution. Furthermore, the area balance encompasses all possible line-length solutions. Therefore, where the primary objective of section balancing is the determination of horizontal shortening magnitude, area balancing provides an analytical solution. We use this analytical solution to develop a comprehensive, robust analysis of the uncertainty in shortening estimates resulting from cross-section balancing. The analytical solution allows us to propagate errors formally on the input parameters — stratigraphic thicknesses, depth to decollement, eroded hanging wall cutoffs — through the equations and produce the resulting uncertainty on the magnitude of shortening. Balanced cross sections from the Subandean belt of the Central Andes are used to demonstrate the relative importance of stratigraphy and eroded hanging wall cutoffs in the contribution to the overall error.

1. Introduction

Balanced cross sections have been a fundamental tool of the structural geologist for more than 50 years, providing both a geometric model of the subsurface as well as an estimate of the shortening in a specific region of an orogen. Once derived, shortening magnitudes are often used as input "data" for large-scale geologic models, such as geodynamic models or palinspastic restorations. For example, Kley and Monaldi (1998) use surface shortening estimates in the Central Andes to suggest that crustal thickness cannot be derived from shortening alone, and thus call on underplating or flow of lower crustal material to produce the excess thickening. While this type of analysis may help advance tectonic modeling, these models rely on shortening data that do not include a rigorous assessment of the uncertainty. Without a standardized way to assess the goodness of fit of a specific balanced cross section to the data on which it is based, no independent method exists to determine the validity of conclusions based on shortening estimates from line-length balanced sections.

Though often well known to the structural geologist who constructed the original line-length balanced section, users of the calculated shortening values may overlook the uncertainty inherent in any cross-sectional model as well as the fact that the cross sections are extrapolated from incomplete data. Viable cross sections may follow generalized rules for construction, assuring that the cross section does not violate physical laws such as the continuity and compatibility equations. Such rules, however, do not guarantee that a calculated shortening value has negligible uncertainty.

We present a new method to calculate a rigorous estimate of uncertainty in shortening values from regional line-length balanced sections. This method includes all potential sources of error on input parameters except for the assumption of plane strain deformation. Based on area balancing, the method encompasses all possible kinematic fold-fault models, accommodates shortening due to plane strain deformation smaller than the scale of the cross section, and is computationally simple. By including a full assessment of the uncertainties in a cross section, it is possible to propagate formally the known, measurable uncertainties from the input data through the shortening calculation and determine an uncertainty estimate for the final shortening value.

To demonstrate the application of the concept, we test the method on several cross sections from the Subandean belt of the Central Andes. We compare between blind and emergent thrust belts, as well as sections drawn by the same and different authors. While we describe the results of the formal approach, the primary outcome from this test is to emphasize that the goodness of the calculated uncertainty values depends on the goodness of the initial uncertainties on the input data.

2. Existing methods of cross-section construction

The physical justification for all section balancing methods arises from the continuity equation, which states that the change in

* Corresponding author. Tel.: +1 607 255 3376.
E-mail address: rwa1@cornell.edu (R.W. Allmendinger).

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density of a volume with respect to time plus the flux of mass into and out of the volume must be equal to zero (e.g., Malvern, 1969). In volume balancing, we assume that the change in density with time is also zero, yielding the incompressibility condition, requiring the divergence of the velocity field to be zero. For area balancing, one additional condition is required: plane strain, or the condition that there is flow of material only in the plane of the cross section. This final condition is justified where structures are both long and continuous parallel to the strike, as is true in many thin-skinned fold thrust belts. Specific fold-fault models, including trishear (Zehnder and Allmendinger, 2000), fault bend- and fault propagation-folding (Hardy, 1995, 1997), explicitly use incompressibility, but the majority of balanced cross sections are geometry-specific and therefore more restrictive.

2.1. Line-length balancing

Line-length balanced cross sections are a subset of area-balanced sections. In addition to the assumptions inherent to area balancing, line-length balancing relies on the assumption that parallel folding occurs via shear parallel to bedding, making the stratigraphic horizon lines of no finite longitudinal extension. Thus, the shortening magnitude is the difference between bed length in the deformed state and the same bed in the undeformed state. This method requires a cross-sectional model of the subsurface geometry that tries to replicate the subsurface geology and is governed by generalized empirical rules to help insure viability (Bally et al., 1966; Dahlstrom, 1969; Elliott, 1983; Price and Mountjoy, 1970; Woodward et al., 1989).

Shortening values from line-length balancing are commonly cited as a "minimum estimate", which is typically the only uncertainty referenced. This minimum estimate arises where the hanging wall cutoffs of emergent thrusts in a section have been eroded (Fig. 1). Because the geologist does not know how much bed length is missing due to erosion, the stratigraphic horizons are lined up to make the displacement as small as possible in the restored section.

However, eroded hanging wall cutoffs are only one of the many potential sources of uncertainty in a balanced section and thus "minimum estimate" is misleading (Allmendinger, 2004; Elliott, 1976; Sheffels, 1990). Errors may also arise due to uncertainties in depth to decollement, incorrect structural model of the subsurface, poorly known initial stratigraphy, and deformation at scales smaller than the resolution of the section (e.g., Marrett and Allmendinger, 1992). One possible way to account for variation in shortening from all but the last of these sources of error would be to draft a large suite of line-length balanced sections along the same transect, spanning the range of possible internal geometries and initial conditions in a type of hand-crafted Monte Carlo simulation. Though possible for individual structures amenable to numerical simulation (e.g., Allmendinger, 2004; Brooks et al., 2000), this approach is impractical for regional sections across many structures.

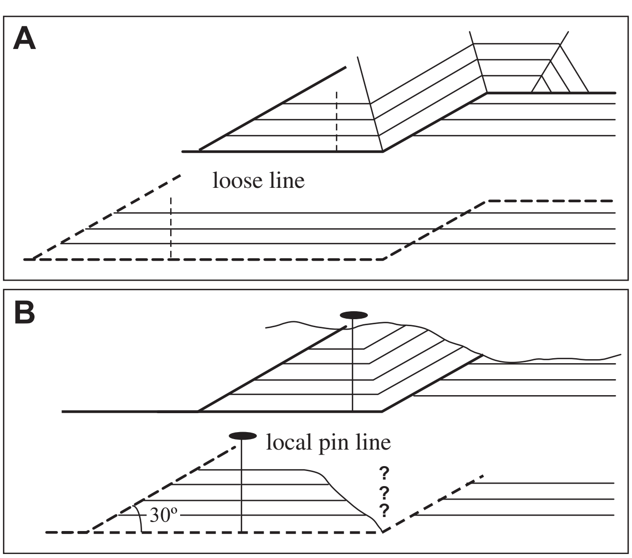

Fig. 1. Cross section illustrating the "minimum shortening estimate" commonly associated with line-length balanced cross sections; (A) shows the actual section in the absence of erosion; (B) shows the same section where the hanging wall cutoff has been eroded. The restoration in the bottom panel of (B) is minimum because the geologist does not know how far to the right the hanging wall ramp would lie.

2.2. Area balancing

Area balancing, based only on the assumption that the cross-sectional area of the modern, deformed thrust belt is equal to the area of the undeformed stratigraphic section (Chamberlin, 1910, 1919b, 1919a, 1923; Hossack, 1979; Mitra and Namson, 1989), is a more generalized method than line-length balancing. Because area balancing provides a method of calculating shortening that does not depend on one specific subsurface geometric interpretation, the method does not provide the geometric or temporal resolution of line-length balancing. However, this independence is also the greatest strength of area balancing: the method encompasses any two-dimensional kinematic fold model that fills the required area and captures all scales of deformation. This attribute of area-balanced cross sections makes them uniquely suited to the task of determining uncertainty in shortening magnitude.

3. Error analysis via area balancing

The horizontal shortening in any balanced section is the difference between the initial and final widths of the section, which is not the same as the principal shortening axis (e.g., Cladouhos and Allmendinger, 1993). For area balancing, we define the initial area as a simple polygon defined by the stratigraphic thicknesses and their uncertainties at each end and the initial width, which is unknown at the start of the calculation (Fig. 2). Unlike the case of line-length balancing, the areas in both the initial and the final (i.e., deformed) state can be calculated analytically. Thus, the errors can be propagated formally, which is a major advantage of this approach. We use the terms "uncertainty" and "error" interchangeably.

3.1. Analytical determination of shortening and error propagation

For the area of the deformed section, we use the concept of an enveloping polygon to encompass the pre-growth strata in the section. The area of any polygon can be described analytically as (e.g., Harris and Stocker, 1998):

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i), \tag{1}$$

where A is the area of the polygon, n is the number of vertices in the polygon, and (x_i, y_i) are the Cartesian coordinates of each vertex. The calculated uncertainty, or error, on the deformed area (δA) is a function of the uncertainty on each specific vertex (δx_i, δy_i) (Fig. 1A). As discussed below, these uncertainties encompass both those associated with eroded hanging wall cutoffs and depth to decollement. By assuming a Gaussian distribution of uncertainty values, we use the standard formula to propagate error through the area calculation (Bevington and Robinson, 2003; Taylor, 1997), known as the sum in quadrature:

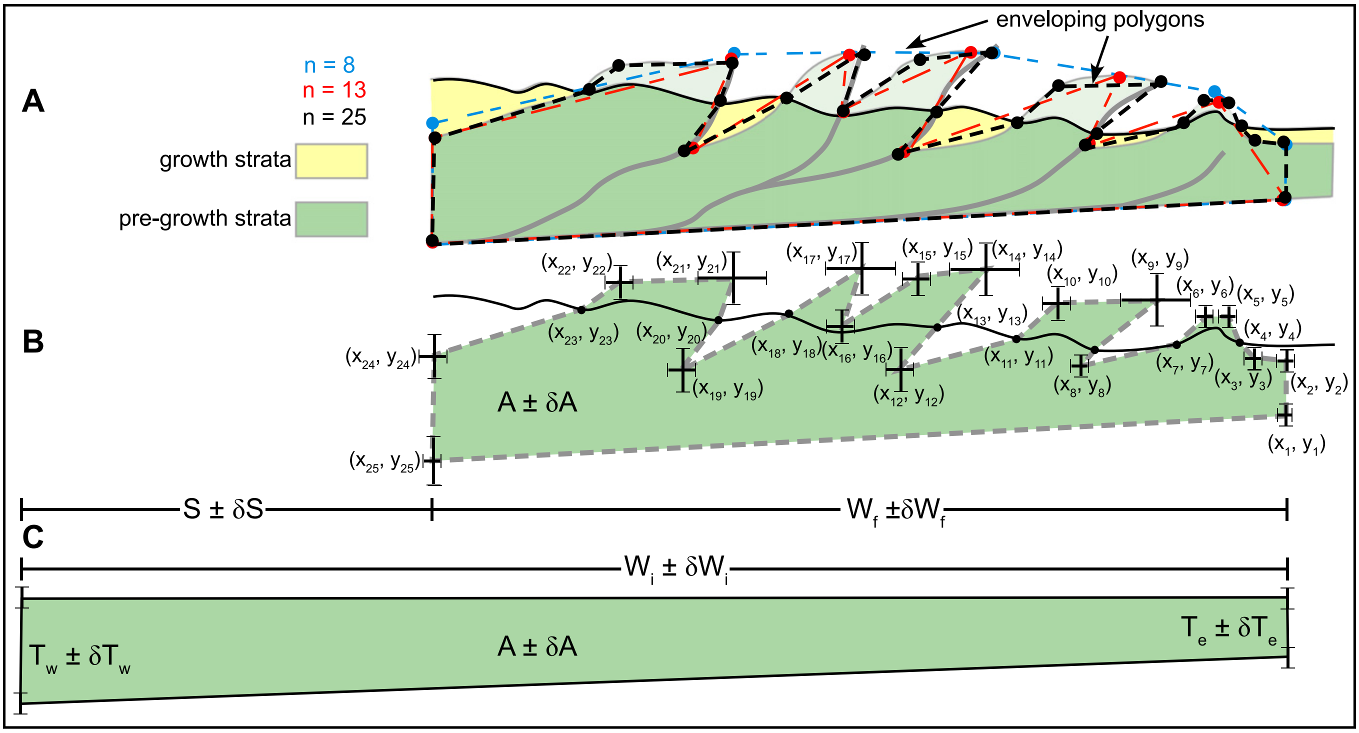

Fig. 2. Hypothetical area balance with attendant uncertainties. Three different "enveloping polygons" are shown in (A) with increasing number of vertices and thus increasing complexity. (B) Assignment of hypothetical uncertainties (error bars) to each of the 25 vertices in the most complex enveloping polygon. (C) The stratigraphic wedge in the initial state, as well as the horizontal shortening. Variables are the same as those used in the text.

$$\delta A = \sqrt{\left(\frac{\partial A}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial A}{\partial y_1}\delta y_1\right)^2 + \cdots + \left(\frac{\partial A}{\partial x_n}\delta x_n\right)^2 + \left(\frac{\partial A}{\partial y_n}\delta y_n\right)^2}. \tag{2a}$$

If the errors are not random and uncorrelated, then one should use, instead, the maximum error estimate:

$$\delta A \leq \left|\frac{\partial A}{\partial x_1}\right|\delta x_1 + \left|\frac{\partial A}{\partial y_1}\right|\delta y_1 + \cdots + \left|\frac{\partial A}{\partial x_n}\right|\delta x_n + \left|\frac{\partial A}{\partial y_n}\right|\delta y_n. \tag{2b}$$

In the rest of this paper, we will show the error in quadrature in equation "a" is accompanied by the maximum error estimate in equation "b".

Because the deformed area must equal the undeformed area, the initial width of the section ($W_i \pm \delta W_i$) can be calculated from the area in Eq. (1) and the two stratigraphic thicknesses at the "west" and "east" ends of the section (T_W, T_E) (Fig. 1A):

$$A = (W_i T_E) + \left(W_i \frac{(T_W - T_E)}{2}\right) = W_i\left(\frac{T_W + T_E}{2}\right). \tag{3}$$

The original width is calculated by rearranging Eq. (3):

$$W_i = \frac{2A}{(T_E + T_W)} = 2A(T_E + T_W)^{-1}. \tag{4}$$

By definition, A in Eqs. (3) and (4) must be the same as A in Eq. (1). The uncertainty for the initial width is:

$$\delta W_i = \sqrt{\left(\frac{\partial W_i}{\partial A}\delta A\right)^2 + \left(\frac{\partial W_i}{\partial T_E}\delta T_E\right)^2 + \left(\frac{\partial W_i}{\partial T_W}\delta T_W\right)^2} \tag{5a}$$

$$\delta W_i = \left|\frac{\partial W_i}{\partial A}\right|\delta A + \left|\frac{\partial W_i}{\partial T_E}\right|\delta T_E + \left|\frac{\partial W_i}{\partial T_W}\right|\delta T_W \tag{5b}$$

where δT_W and δT_E are the uncertainties on stratigraphic thicknesses and δA is the area error calculated in Eq. (2).

Finally, knowing the initial width ($W_i \pm \delta W_i$) and the final, deformed width ($W_f \pm \delta W_f$) (Fig. 1) allows us to calculate the shortening, S, and its uncertainty, δS, across the fold and thrust belt:

$$S = W_f - W_i \tag{6}$$

and

$$\delta S = \sqrt{\left(\frac{\partial S}{\partial W_i}\delta W_i\right)^2 + \left(\frac{\partial S}{\partial W_f}\delta W_f\right)^2}. \tag{7a}$$

$$\delta S = \left|\frac{\partial S}{\partial W_i}\right|\delta W_i + \left|\frac{\partial S}{\partial W_f}\right|\delta W_f \tag{7b}$$

We can also calculate the percent horizontal shortening and its error:

$$S_{\%} = 1 - \frac{W_f}{W_i} = 1 - W_f W_i^{-1}, \tag{8}$$

and

$$\delta S_{\%} = \sqrt{\left(\frac{\partial S_{\%}}{\partial W_i}\delta W_i\right)^2 + \left(\frac{\partial S_{\%}}{\partial W_f}\delta W_f\right)^2} \tag{9a}$$

$$\delta S_{\%} = \left|\frac{\partial S_{\%}}{\partial W_i}\right|\delta W_i + \left|\frac{\partial S_{\%}}{\partial W_f}\right|\delta W_f. \tag{9b}$$

To calculate the uncertainty in shortening magnitude and percentage, errors must be specified for the input parameters: the vertices of the enveloping polygon, the stratigraphic thicknesses at the two ends of the line of section, and the deformed width of the thrust belt (Fig. 2). The errors for the enveloping polygon, δx_i and δy_i, encompass both the uncertainties in the depth to the decollement and those associated with any eroded hanging wall cutoffs. Independent uncertainties are assigned to each vertex such that contacts at the surface generally have negligible error but those in the subsurface have greater error, and those at eroded hanging wall cutoffs have the greatest uncertainty of all. The errors on the stratigraphic thicknesses, δT_E and δT_W, would ideally come from measured sections where available, but more commonly will come from map thicknesses, which would likely have larger errors. All other sources of error are propagated from δx_i, δy_i, δT_E, and δT_W (Fig. 1A).

3.2. Minimal polygon complexity necessary to capture accurate shortening

Using polygons to envelop the cross-sectional area raises the important question of how complex the polygon must be to capture accurately the shortening for the region. One can imagine two extremes: a simple rectangle enclosing the entire deformed area or a very complex polygon with hundreds of vertices that replicates the outline of the specific line-length balanced cross section. Between these two cases lies an ideal polygon that captures the minimum complexity needed to calculate a robust, stable area estimate but is not heavily reliant on the modeled subsurface geometry. While a polygon with 5 vertices is clearly a poor estimate of the subsurface geology (Fig. 2), a polygon with 75 vertices would likely be overly restrained by the originally proposed line-length balanced model.

To determine the minimum number of vertices necessary for a robust shortening calculation, we iterate the analysis with increasingly complicated enveloping polygon geometries (Fig. 2) until the solutions for both the shortening magnitude and uncertainty stabilize (Fig. 4). For the Subandean test cases described in the subsequent section of this paper, the shortening solutions stabilize for polygons of approximately 20 or more vertices, far fewer than needed to capture

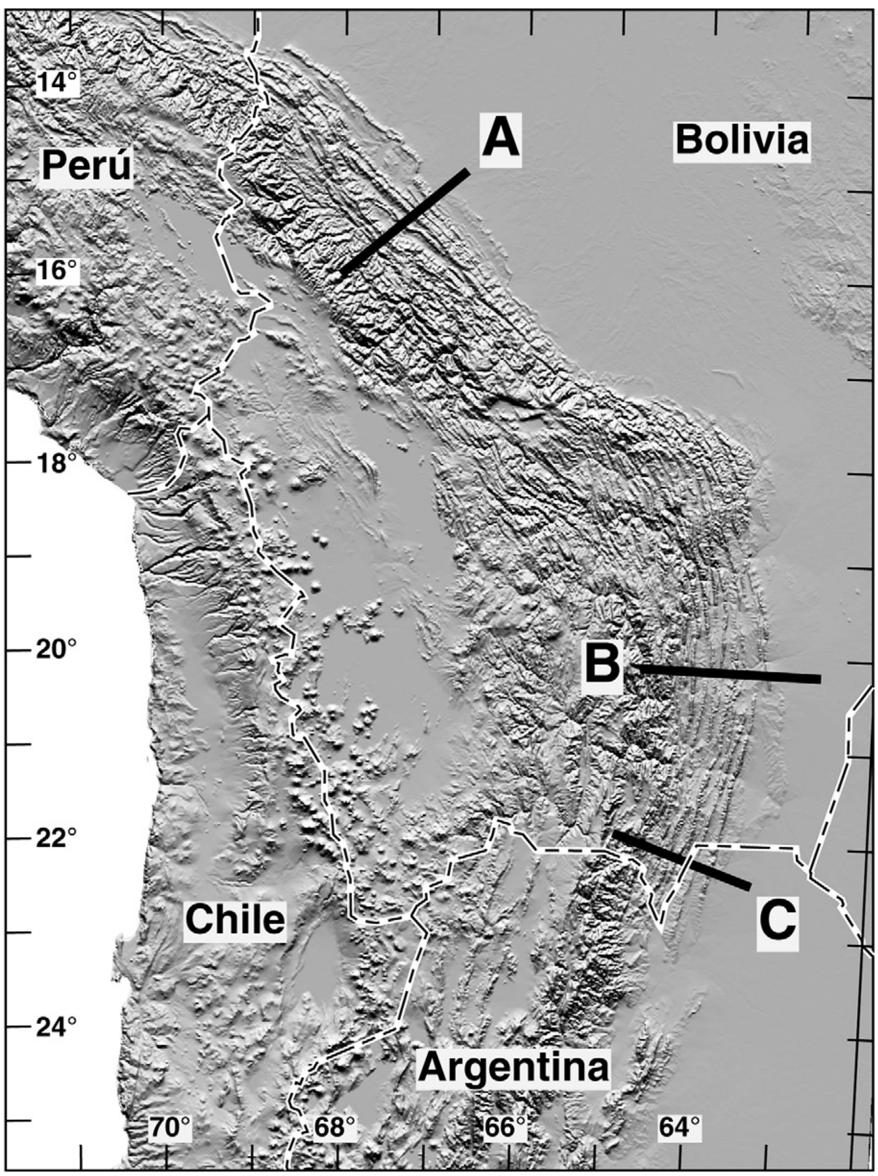

Fig. 3. Location map of the Central Andes showing the balanced cross-sections analyzed from the Subandean belt. Sections A and B, located in Bolivia, were published and described by McQuarrie (2002) and McQuarrie et al. (2008); section C, located in northernmost Argentina, was published by Echavarría et al. (2003). Shaded relief topography rendered from the GTOPO30 data set.

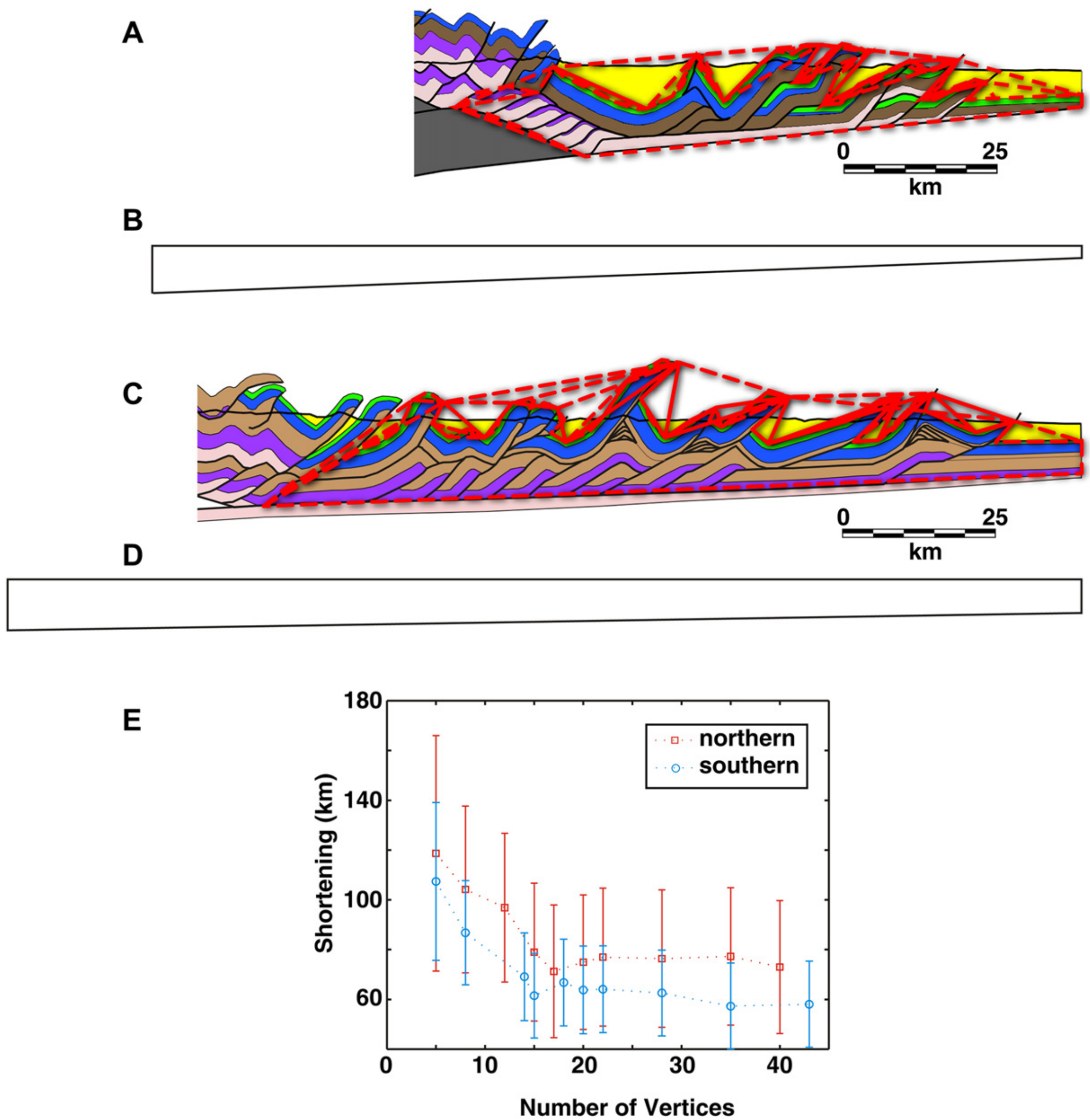

Fig. 4. (A) Cross section from the northern Bolivian Subandean belt, modified after McQuarrie (2002). The red dashed lines show the suite of enveloping polygons used in the analysis. (B) The initial stratigraphic wedge for the northern section. (C) Geologic cross section for the southern Bolivian Subandean Belt, modified after McQuarrie (2002). The red dashed lines show the suite of enveloping polygons used in the analysis. (D) The initial stratigraphic wedge for the southern Bolivia section. (E) Plot of number of polygons in the enveloping polygon versus horizontal shortening magnitude in kilometers for both the northern and the southern sections of McQuarrie (2002). Error bars show the uncertainty at each point in the analysis; note overlap of error bars for northern and southern sections. The solution stabilizes at about 20 vertices. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the exact outline of the line-length balanced section on which the polygons are based. This stability is likely due to compensating errors: after 15 or 20 vertices, adding another vertex, with its associated uncertainty, does not significantly change the overall solution. However, it is true that some crude approximation of the line-length section is also necessary, reflecting the fact that the polygons with fewer vertices invariably include a significant amount of growth strata or air that was never filled with subsequently eroded rock.

4. Test cases from the Subandean belt

To demonstrate the application of our area balancing method, we use three sections from the Subandean belt of Bolivia and northern Argentina (Fig. 3), two dominated by emergent thrusts (McQuarrie, 2002; McQuarrie et al., 2008) and the other blind (Echavarría et al., 2003). These three sections allow us to compare the results of sections drawn by the same authors (McQuarrie, 2002; McQuarrie et al., 2008) and to compare between different authors (Echavarría et al., 2003; McQuarrie, 2002). The quality of our error analysis depends on using reliable uncertainties on the input data; one of the authors of this paper (RWA) was involved with the construction of the Echavarría cross section and Nadine McQuarrie (pers. comm., 2010) has graciously shared her insight on the uncertainties involved in the construction of her sections. We show the input parameters and uncertainties that we used in our analysis in Table 1 and the shortening results based on those values, compared to previous work, in Table 2. However, the best practice, as described below, is a rigorous assessment of errors on the input parameters during construction of cross-sections. Thus the results presented in these test cases should be viewed as a proof of concept.

Table 1
Reference case inputs.

	Northern Bolivia	Southern Bolivia	Northern Argentina
Stratigraphic thickness & errors	2.1 ± 0.8 km 8.10 ± 1.2 km	5.6 ± 0.8 km 8.5 ± 0.8 km	2.9 ± 0.6 km 4.6 ± 0.4 km
Final (Modern) width & errors	96 ± 1 km	113 ± 1 km	82 ± 1 km
Decollement error	±0.75 km	±0.75 km	±0.5 km
Subsurface vertices error	±0.8 km	±0.8 km	±0.6 km
Surface vertices errors	±0.1 km	±0.1 km	±0.1 km
Eroded hanging wall errors	±3.0 km	±3.0 km	±1.0 km

Table 2
Comparison of previous line-length balancing with area balancing.

	Northern Bolivia	Southern Bolivia	Northern Argentina
Line-length balance (McQuarrie, 2002; Echavarría et al., 2003)	66 ± 7 km 41 ± 2%	67 ± 7 km 32 ± 2%	45 km
Area balance ± Gaussian error (this study)	75 ± 27 km 44 ± 9%	64 ± 17 km 36 ± 6%	48 ± 15 km 37 ± 7%
Area balance ± maximum error (this study)	75 ± 72 km 44 ± 24%	64 ± 49 km 36 ± 18%	48 ± 44 km 37 ± 22%

4.1. *Bolivian sections*

Our area balancing method yields 75 ± 27 km of shortening when applied to the northern Bolivia section and 64 ± 17 km for the southern section (Table 2, Fig. 4). Our estimates do not include shortening on the trailing thrusts in the sections. These uncertainties on shortening have been calculated based on input error values for the deformed width, the location of each polygon vertex, and for the stratigraphic thicknesses in the undeformed state (Table 1). To examine the effect that a single parameter (e.g., stratigraphic thickness, decollement depth, or eroded hanging wall cutoffs) has on the total uncertainty, we set all errors, except for the parameter of interest, equal to zero and then ran the analyses over again (Table 3). Note that total error should generally be less than the sum of the individual errors for a Gaussian distribution. If the input uncertainties are not independent and uncorrelated, then the errors no longer have a Gaussian distribution and one would use the maximum error estimate, which is considerably larger (Table 2).

Percent shortening is a more ambiguous measure because it is so highly dependent on initial and final lengths. Nonetheless we cite them here because McQuarrie et al. (2008) claimed there was a significant difference between percent shortening in the northern and southern cross sections. As shown in Table 2, the shortening percentage values for the two regions in Bolivia are similar to those calculated via line-length balancing. This is not surprising given that we used McQuarrie's sections as the starting point for our area analysis. However, the errors that we calculate are three to five times larger than the 2% error cited by McQuarrie et al. (2008). The considerable overlap in error envelopes for the two sections (Fig. 4) shows that their conclusion of a coupling between tectonics and climate based on similar shortening percentages is not robust.

Table 3
Contributions to total error.

Error	Northern Bolivia	Southern Bolivia	Northern Argentina
Total	±27 km	±17 km	±15 km
From stratigraphic thickness	±24 km	±16 km	±12.5 km
From decollement	±7 km	±1 km	±1.1 km
From eroded hanging wall	±2 km	±4 km	±1.2 km

4.2. *Argentine section*

To analyze the differences in uncertainty values for sections dominated by blind thrusts as compared to emergent thrusts, we calculate shortening and uncertainty values for the section in northern Argentina (Echavarría et al., 2003). For simplicity, we balance the section to the east of the Nogalito Range (Fig. 5) because, west of this range, the section, as drawn, cuts into the basement thrust. We calculate 48 ± 15 km shortening for the eastern part of the section, compared to 45 km shortening determined via line-length balancing (Echavarría et al., 2003). As in the case of the Bolivian sections, Table 3 shows the effect that each parameter has on the overall error.

The shortening and uncertainty results from area balancing for the two southernmost sections are very similar: 36 ± 6% for the Bolivian section (McQuarrie et al., 2008) and 37 ± 7% for the northern Argentine section (Echavarría et al., 2003). The percent shortening values for both sections agree with those calculated via line-length balancing, and it is the similarity between the uncertainty values that is noteworthy. While the region in Argentina and southern Bolivia is well studied, with both seismic and drill hole data available (Belotti et al., 1995; Dunn et al., 1995; Sempere, 1995), the section in northern Bolivia is not as well known. However, we use similar initial uncertainty values for all three sections to demonstrate the method and not to determine definitively the uncertainty associated with each section.

4.3. *Sensitivity of total error to different parameters*

As is abundantly clear from these Subandean examples, the error on stratigraphic thickness is a major source of shortening uncertainty. In all sections, 8–40% error in stratigraphic thicknesses on the two ends of a cross section accounts for 80% or more of the total error in shortening determination (Table 3). Even for a reasonably well known section, a 10% uncertainty in stratigraphic thickness contributes 50–75% of the overall shortening error. If one's only objective were to calculate shortening, significantly greater reduction in errors could be achieved through field studies necessary to improve knowledge of stratigraphic thickness than one could produce by carrying out a much more expensive program of subsurface exploration. Granted, other advantages exist to collecting data from subsurface exploration, especially the knowledge of subsurface geometry!

The relative importance of decollement depth and eroded hanging wall cutoff depends on the specific sections (Table 3). In a fully emergent belt with a large number of eroded hanging wall cutoffs, the contribution of this factor to the overall uncertainty would increase. In line-length balancing, because one determines the initial width simply by adding up the lengths of individual beds, the uncertainty in hanging wall cutoff would translate directly into shortening magnitude uncertainty. This is not the case in area balancing. For example, the Argentine section has one major eroded hanging wall cutoff, changing the uncertainty in that cutoff from 1 to 5 km only changes the uncertainty in shortening by 1 km (from ±15 km to ±16 km) for the reference case.

5. Accurate determination of errors on input parameters

As the test cases show, accurate determination of input errors is critical. Our method does not alleviate this task but only makes it clear what the key parameters are and, once determined, how to

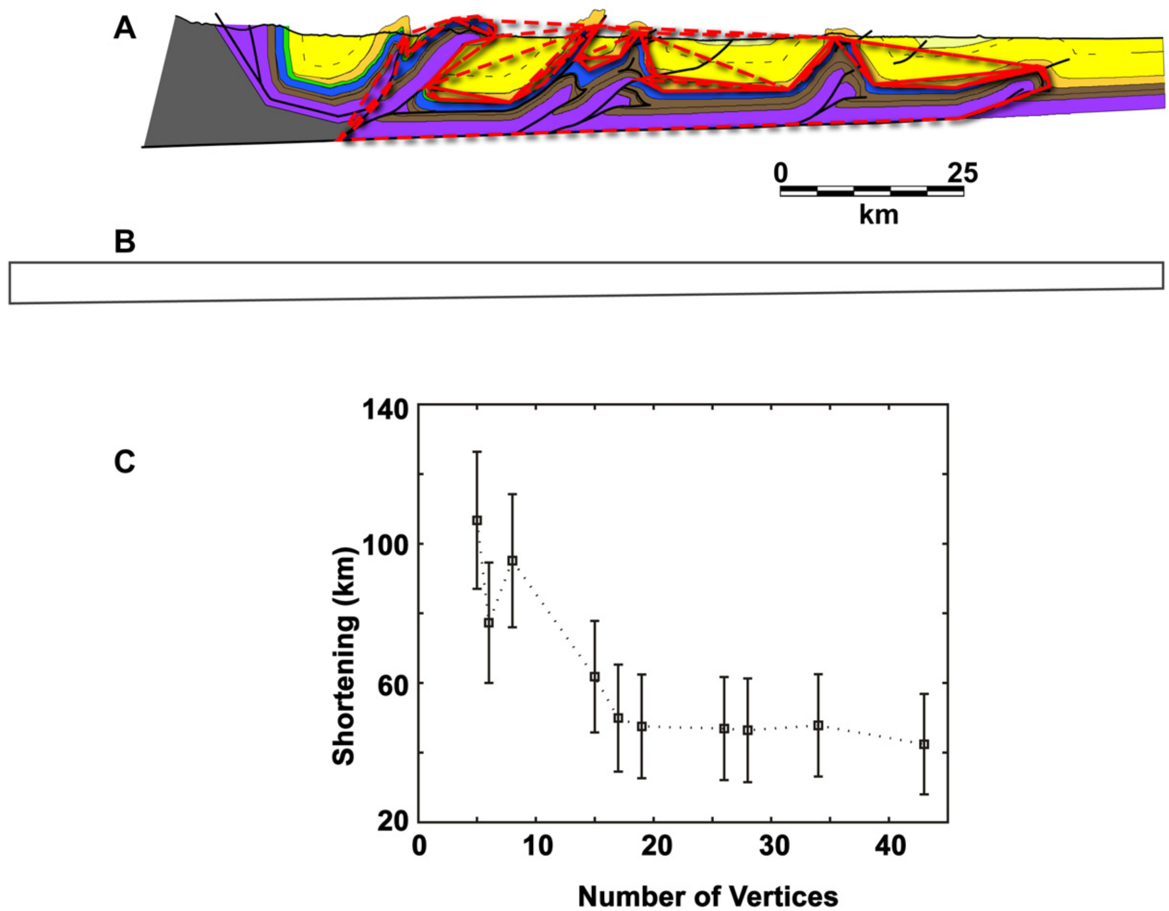

Fig. 5. Diagram similar to Fig. 4, showing the results of the analysis of Echavarría et al.'s (2003) cross section from the northern Argentine Subandean belt. (A) Geologic cross section for the southern Bolivian Subandean Belt, modified after Echavarría et al. (2003). The red dashed lines show the suite of enveloping polygons used in the analysis. (B) The initial stratigraphic wedge for the section. (C) Plot of number of polygons in the enveloping polygon versus horizontal shortening magnitude in kilometers. Error bars show the uncertainty at each point in the analysis. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

propagate those errors through the calculation to produce realistic errors for shortening magnitude.

5.1. *Deformed state: the enveloping polygon*

Assigning uncertainty to each of the vertices in the enveloping polygon accounts for a number of different types of geological errors. While there is uncertainty inherent to choosing a specific fold kinematic model, an area balancing method eliminates this uncertainty by accommodating all cylindrical fold models that can occupy the same area.

The vertices at the base of the polygon describe the position and uncertainty in the decollement depth and dip. The errors assigned to these vertices would depend on the source and quality of data — borehole, reflection seismic, and stratigraphic — that the geologist used to identify the decollement. If stratigraphic data are used to define the decollement depth, then the assumption that the errors are random and uncorrelated would not be valid, requiring the use of the maximum error estimate (e.g., Table 2) rather than assuming a Gaussian distribution. Other errors on subsurface vertices would likewise depend on the availability and quality of subsurface data. For example, is a broad syncline of growth strata imaged clearly on seismic data or is it pierced by a well? Depending on the placement of each vertex and the quality of the available data, the uncertainty associated with a specific polygon could be quite variable.

The errors on the vertices that lie above the present erosional surface present different challenges. Where the faults are largely blind, one can geometrically project the crest of an eroded anticline based on stratigraphic thickness and some basic assumptions of fold kinematic model (or range of models). The largest individual uncertainties on vertices in the deformed state probably are those associated with eroded hanging wall cutoffs, though they do not contribute that much to the overall uncertainty. Although these are accommodated via minimum shortening estimates in line-length sections, there is a reasonable maximum projection of the hanging wall cutoff as well. If the section of interest lies close to the tip line of an emergent thrust, one could use an up-plunge projection and some model of the displacement gradient profile along a fault (Higgs and Williams, 1987; Marrett and Allmendinger, 1990; Walsh and Watterson, 1987, 1989) to determine where the now eroded hanging wall cutoff should lie. More commonly, we suspect, people will use their intuition as to the probable location of the cutoff and simply select a large uncertainty.

5.2. *Initial state: the stratigraphic wedge*

As we have seen, uncertainty in initial stratigraphic thickness is a major source of error that is rarely included in line-length sections. To estimate this uncertainty, as well as improve the overall shortening estimate, one might measure several stratigraphic sections at either end of the now deformed package. Alternatively, because data on balanced cross sections are commonly projected from a corridor of finite width on either side of the section, one might use the variation in map thickness that occurs along strike within that corridor.

As mentioned previously, one source of error in balanced sections is the shortening that occurs at scales below the resolution of the section (Marrett and Allmendinger, 1990, 1991, 1992). This might include initial layer parallel shortening, pervasive minor faulting and folding, pressure solution and cleavage development, etc. (e.g., Geiser, 1988; Groshong and Epard, 1994; Groshong, 1994; Hossack, 1979). Hypothetically, simply by doing an area balance, we capture this deformation as well. However, the ability to do so depends on one's ability to determine true initial stratigraphic thickness prior to the start deformation. Because we measure stratigraphic sections today, in the deformed state, it is much more difficult to ensure that deformation due to pervasive mechanisms has not been included in our determination of stratigraphic thickness.

Pressure solution and cleavage development are commonly associated with volume loss, bringing into question the plain strain and constant volume or area assumptions. Area loss balanced by area gain in another part of the section presents no particular challenges to the error propagation method described here. An average net area loss from the entire section could easily be accommodated by adding a term expressing that average area loss (and uncertainty) to Eq. (2). It would be very difficult to determine an accurate average area loss, however.

5.3. What is the true magnitude of the shortening?

Determining a single value of shortening magnitude for a belt is somewhat ambiguous and arbitrary, and percent shortening is even more fuzzy. Take the case of McQuarrie's section in southern Bolivia (McQuarrie, 2002; McQuarrie et al., 2008). The regional pin lines are located 10–15 km east of the thrust front, which reduces the percent shortening by inclusion of a significant width of undeformed section, but does not affect the magnitude of shortening. More subtle, but equally important, in a line-length balanced section, different stratigraphic horizons can have different shortening magnitudes because of internal duplexing of some layers but not others (McQuarrie, pers. comm., 2010). Where one knows the structural geometry a priori, this could be very important because the initial undeformed polygon would not be a simple wedge as we have portrayed it but a more complicated polygon, with multiple steps on the internal side (Fig. 6). However, we usually do not know the structural geometry ahead of time and duplexes are commonly used to accommodate space problems that may actually arise from poorly known stratigraphy. Finally, the present day width of a belt is commonly determined by its width at the surface, but its maximum width at depth is longer because of the dip of the trailing thrust. McQuarrie (2002) avoided this ambiguity by defining the Subandean belt by the basement cutoff. This definition is not without problems — McQuarrie et al. (2008) use a different and more traditional definition — both because we do not know the location of that cutoff, and because it results in provinces that overlap (i.e., the eastern boundary of the Interandean belt lies east of the western boundary of the Subandean belt). Using one measure rather than the other can change the magnitude of shortening by many kilometers, even though the shortening error does not change much because the uncertainty on final width contributes little to the overall error.

5.4. Can we determine true probabilistic uncertainties?

Ideally, one would like to be able to state the shortening at, the alpha-95 confidence level, for example. The error propagation that we describe here would allow for this but the real question is whether the input data allow for the determination of true probabilistic uncertainties, which depends on repeated measurements of the same parameter. While one can imagine approaches to determine the one or two sigma errors on the depth to decollement or the stratigraphic thickness at one end of the cross section or the other, these approaches would probably require more effort than most people have traditionally put into balanced cross-section construction.

What, then, is the advantage of carrying out error propagation if the probabilistic errors will not routinely be determined? Most importantly, it is the best way of quantitatively linking the uncertainty on the input parameters, even if determined only informally, to the likely error on the shortening. More specifically, error propagation provides a mechanism for investigating the effects of different types of uncertainty on the final solution. Most obvious is the previously under-appreciated effect of stratigraphic thickness,

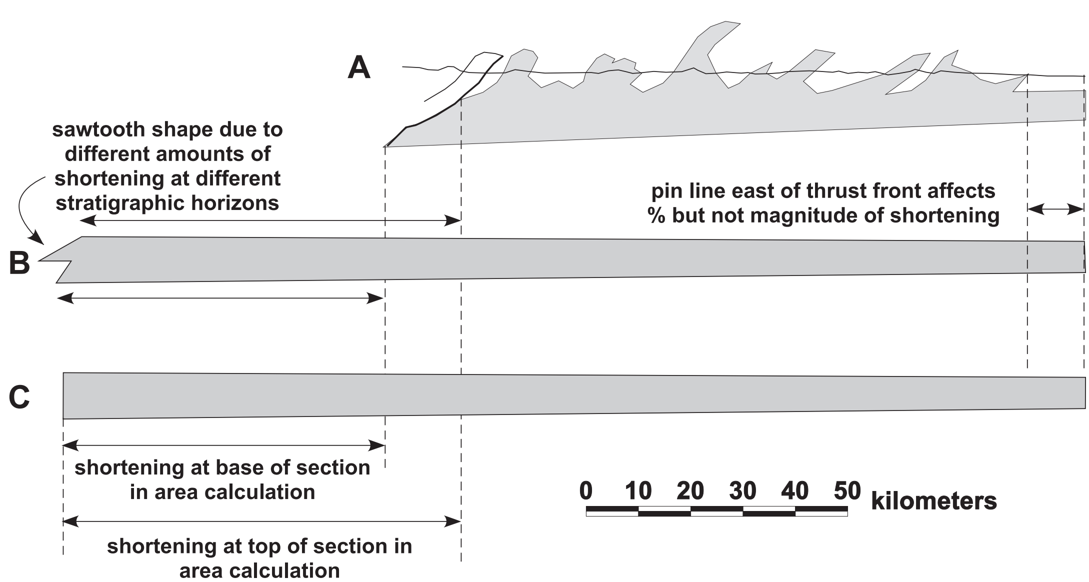

Fig. 6. Illustration of the ambiguities of the shortening magnitude calculation. (A) Outline of the deformed pre-growth strata for the southern Bolivia cross section (McQuarrie, 2002). (B) Outline of McQuarrie's (2002) line-length reconstruction of the section in (A). Note the sawtooth left side of the section is due to different amounts of shortening at different stratigraphic horizons. (C) The equivalent area balance of the deformed gray polygon shown in (A). Horizontal double headed arrows show different permissible values of shortening magnitude.

but this also applies to the relative importance of uncertainty in decollement position or eroded hanging wall cutoffs for any particular section. Finally, it provides a mechanism for specifying where one's preferred line-length solution lies within the range of plausible solutions due to different types of fold kinematic models.

6. Conclusions

Calculating the magnitude of shortening in a mountain belt is the end result of a structural model that is constructed from data that have quantifiable errors. Without propagating these errors through the analysis, structural geologists have no scientifically legitimate way of determining whether two parts of an orogen have distinct shortening values and therefore that external processes, climate, or plate boundary interactions explain those differences. Likewise, other uses of structural shortening data — geodynamic modeling, paleogeographic reconstructions, etc. — are equally suspicious if the uncertainty on their input data, the shortening value, cannot be quantified accurately.

Line-length balanced sections with errors assessed via area balancing are entirely complementary. Line-length balanced sections make predictions in the form of detailed geometric models of the subsurface, which can be tested and refined. The internal structural models are particularly useful for identifying and assessing potential sources of subsurface resources or the sequences in which the structures developed. Nonetheless, the practice of using only one line-length balanced section to calculate shortening and using only the hanging wall cutoffs to estimate uncertainty is flawed when the primary objective is a thorough estimate of orogenic shortening.

The method we have presented here is only the first step in producing a complete analysis of errors in shortening magnitude. Future improvements will account for the considerable likelihood that the initial stratigraphic geometry is more complicated than a simple wedge-shaped foreland basin. Additionally, a scheme to include basement thrusts, internal pinch-outs, and preexisting deformation would allow for the analysis of more regions. Finally, the natural progression of this work would be to expand the method into three dimensions and calculate shortening estimates by volume balancing.

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